as those formed when the propellant is decomposed.[19]

Recently, it has been observed by Glazkova[20] that the deflagration rate of AP is inhibited when AP is mixed with compounds that inhibit AP decomposition. This effect becomes insignificant beyond 3000 psi, but in the pressure range of interest in rocket motors the data show two- to threefold change. Boggs et al.[21] have observed that, within certain dopant concentration ranges and pressure below 2000 psi, the deflagration rate of AP increases as a function of the dopant concentration. The above-mentioned features indicate the thermal decomposition of the propellant and that of AP to be related to the burning rate of the propellant and AP, respectively.

References

[1] Jacobs, P.W.M. and Whitehead, H.M., "Decomposition and Combustion of Ammonium Perchlorate," *Chemical Reviews*, Vol. 69, 1969, pp. 551-590.

[2] Hall, A.R. and Pearson, G.S., "Ammonium Perchlorate: A Review of its role in Composite Propellant Combustion," RPE Technical Rept. No. 67/1, Ministry of Technology, London, Jan. 1967.

[3] Boggs, T.L., Zurn, D.E., and Cordes, H.F., "The Combustion of Ammonium Perchlorate and Various Inorganic Additives," AIAA Paper 75-233, 1975.

[4] Burnside, C.H., "Correlation of Ferric Oxide, Surface Area and Propellant Burning Rate," *Chemical Abstracts*, Vol. 84, 1976, 62005 Y.

[5] Rastogi, R.P., Kishore, K., and Singh, G., "Combustion of Polystyrene and Oxygen-Styrene Copolymer/Ammonium Perchlorate Propellants," *AIAA Journal*, Vol. 12, Jan. 1974, pp. 9-10.

[6] Reich, L. and Stivala, S.S., *Elements of Polymer Degradation*, McGraw Hill Book Company, New York, 1971, p. 115.

[7] Bakhman, N.N., Nikiforov, V.S., Avdyunin, V.I., Fogelzang, A.E., and Kichin, Yu. S., "Catalytic Effect of Ferrous Oxide on Burning Rate of Condensed Mixtures," *Combustion and Flame*, Vol. 22, 1974, pp. 77-87.

[8] Zhalnin, V.V., Bakhman, N.N., Lukashenya, G.V., and Kichin, Yu. S., "Effect of Aluminum and Iron Oxide on the Temperature Coefficient of the Burning Rate of Condensed Mixtures," *Combustion Explosion and Shock Waves* (English translation), May 1973, pp. 374-376.

[9] Evdokimov and Bakhman, N.N., "Effect of Carbon Black and Fe_2O_3 on the Rich Combustion Limit of Condensed Mixtures," *Combustion Explosion and Shockwaves* (English translation), 1969, pp. 403-404.

[10] Shidlovskiy, A.A. and Volodina, N.A., "Study of the Combustion of Potassium Chlorate Isitol Mixtures with Catalytic Additives," *Zhurnal Prikladnoy Khimii*, Vol. 39, 1966, pp. 754-758; also Aerospace Technical Division Rept. 66-68, Aug. 1966.

[11] Shadman-Yazdi, F. and Petersen, E.E., "The Effect of Catalysts on the Deflagration Limits of Ammonium Perchlorate," *Combustion Science and Technology*, Vol. 5, 1972, pp. 61-67.

[12] Friedman, R., Nugent, R.C., Rumbel, K.E.. and Scurlock, A.C., "Deflagration of Ammonium Perchlorate," *VIth Sumposium (International) on Combustion*, 1956 (publ. 1957), pp. 612-618.

[13] Handley, J.C. and Strahle, W.C., "Behavior of Several Catalysts in the Combustion of Solid Propellant Sandwiches," *AIAA Journal*, Vol. 13, Jan. 1975, pp. 5-6.

[14] Jones, H.E. and Strahle, W.C., "Effect of Copper Chromite and Iron Oxide Catalysts on AP/CTPB Sandwiches, *XIV Symposium (International) on Combustion*, 1972 (publ. 1973), pp. 1287-1295.

[15] Pittman, C.U., Jr., "Location of Action of Burning Rate Catalyst in Composite Propellant Combustion," *AIAA Journal*, Vol. 7, Feb. 1969, pp. 328-334.

[16] Singh, G., "Studies on Combustion of Composite Solid Rocket Propellants," Ph.D thesis, University of Gorakhpur, 1974.

[17] Solymosi, F. and Revesz, L., "The Thermal Decomposition of Ammonium Perchlorate in the Presence of Iron Oxide," *Kinetics and Catalysis* (English translation), Vol. 4, 1963, pp. 73-79.

[18] Pai Verneker, V.R., Kishore, K., and Krishna Mohan, V., "Correlation Between Combustion and Decomposition of Solid Propellants," *AIAA Journal*, Vol. 13, Oct. 1975, pp. 1415-1416.

[19] Kishore, K. Pai Verneker, V.R., Chaturvedi, B.K., and Gayathri, V., "Mechanistic Studies on Composite Solid Propellants," *AIAA Journal*, Vol. 15, Jan. 1977, p. 114.

[20] Glazkova, A.P., "Three Possible Ways to Inhibit the Ammonium Perchlorate Combustion Process," *AIAA Journal*, Vol. 13, April 1975, pp. 438-442.

[21] Boggs, T.L., Price, E.W., and Zurn, D.E., "The Deflagration of Pure and Isomorphously Doped Ammonium Perchlorate," *XIIIth Symposium (International) on Combustion*, 1970 (pub. 1971), pp. 995-1008.

Structure of a Radially Directed Underexpanded Jet

J. H. T. Wu,* M. N. Elabdin,† and R. A. Neemeh‡
McGill University, Montreal, Quebec, Canada
and
P. P. Ostrowski§
University of Maryland, College Park, Md.

THE performance characteristics of conventional aerodynamic resonator cavities are influenced greatly by the structure of the underexpanded choked jets that are used to excite them.[1] For pressure ratios less than 4.5 in air, no Mach disk is present, and the exciting jet has a repetitive, cellular structure.[2] Resonance is achieved only when the cavity mouth is located where the pitot pressure increases in the downstream direction, that is, in the downstream half of each jet cell. A 3-in.-diam cylindrical resonator,[3] excited by a circumferentially located and radially directed underexpanded choked jet, has reached its peak performance at a pressure ratio of 3.76. The aim of the present work is a supplement of Ref. 3 for examining more closely those properties of such a jet which affect the performance of this type of resonator.

Calculations are performed according to the method of characteristics for axisymmetric, irrotational flow. In terms of the flow angle θ, Mach angle α, and Prandtl-Meyer angle ω, the slopes of the characteristics and the compatability relations are

$$\frac{dr}{dx} = \tan(\theta \pm \alpha) \tag{1}$$

$$d\omega \mp d\theta - \frac{\sin\alpha \, \sin\theta}{\cos(\theta \pm \alpha)} \frac{dx}{r} = 0 \tag{2}$$

From these relations, the jet structure was calculated. The theoretical results are compared to experimental measurements obtained from schlieren photography and a cone-probe traverse in the jet along the centerline in the radial direction.

A cross-sectional view of the nozzle configuration[3] is shown in Fig. 1. A toroidal plenum chamber with a rectangular cross section and outer diameter of 7.5 in. is fitted with five equally spaced compressed air inlets. A narrow slit of width $W = 0.125$ in. with a ¼-in. rounded entrance in the inner wall serves as a nozzle for the jet which discharges to the atmosphere and is directed radially toward the center of the toroid, for which the radius $R = 1.5$ in.

Figure 2 shows a typical schlieren photograph of the jet structure for a jet pressure ratio of 3.04 and with the optical

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*Associate Professor, Department of Mechanical Engineering.

†Research Engineer.

‡Research Associate and Assistant Professor, Department of Mechanical Engineering, Concordia University, Montreal.

§Assistant Professor, Department of Mechanical Engineering.

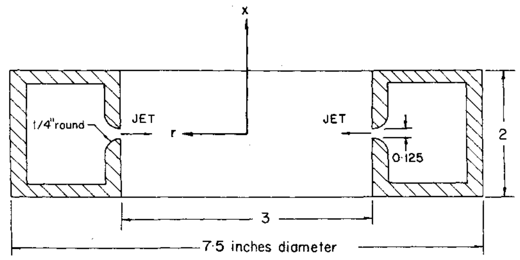

Fig. 1 A cross-sectional view of the 3-in.-i.d. toroidal nozzle as in Ref. 3.

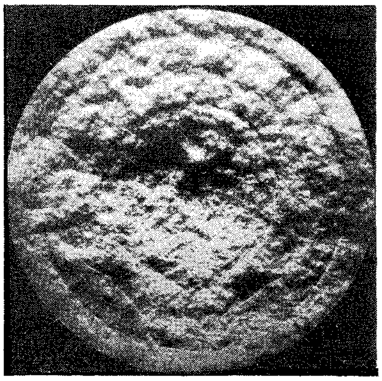

Fig. 2 Schlieren photograph of the jet structure for a jet pressure ratio of 3.04, $R=1.5$ in., and $W=0.125$ in.

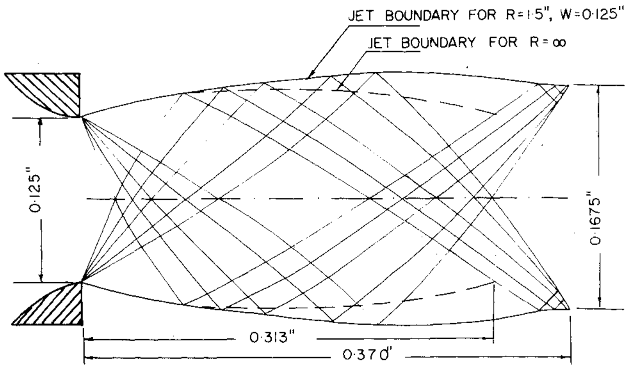

Fig. 3 First cell of the jet structure computed by the method of characteristics at $R=1.5$ in. for a jet pressure ratio of 3.76.

axis coincident with the x axis. In Fig. 2, jet cells are clearly visible as circular rings exhibiting a fair degree of symmetry which progressively deteriorates due to turbulent diffusion and collision of the jet in the central region of the cavity. Figure 2 also illustrates that jet cell length increases with decreasing radius due to flow area convergence.

A theoretical calculation of the first cell of the jet structure is presented in Fig. 3 for a pressure ratio of 3.76. For purposes of comparison, the jet boundary of a two-dimensional jet also is shown. The distinguishing features of the radially directed jet are that the initial jet width is increased by about 34% at the end of the first cell and that the cell length is longer than that of the two-dimensional jet by about 18%. Both result from the conservation of mass flow in the radial direction. Consequently, the radially directed jet cannot possess a truly repetitive structure similar to that which ideally exists in the two-dimensional case.

The variation in first cell length with jet pressure ratio measured from the schlieren photographs is given in Fig. 4. Experimental measurements also are included for a two-dimensional jet issuing from a plane slot 2.0 in. long and 0.125 in. wide. From Fig. 4, it can be seen that the experimental results are very well predicted by the characteristics solution for both the radially directed and two-

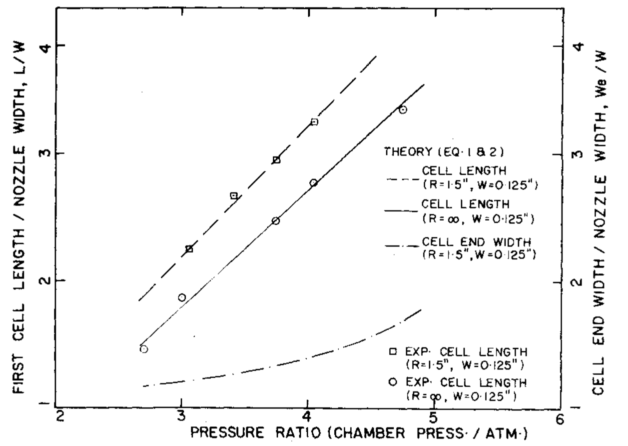

Fig. 4 First cell length L and cell end width W_e vs jet pressure ratio for the nozzle width $W=0.125$ in.

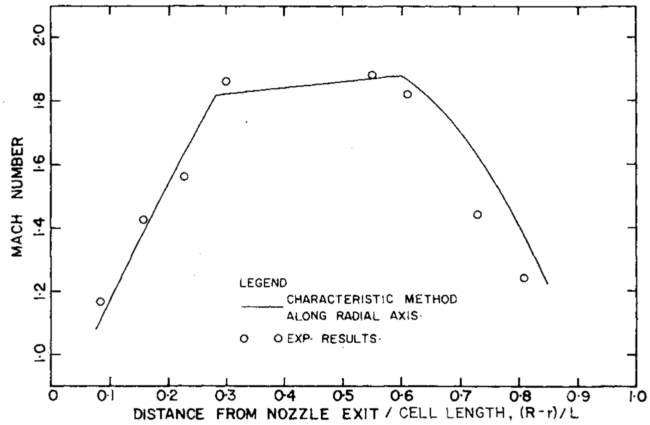

Fig. 5 Mach number, distribution in the first cell for a jet pressure ratio of 3.76, $R=1.5$ in. and $W=0.125$ in.

dimensional jets over the pressure range of the tests. Also in Fig. 4, the characteristics solution shows that the width W_e at the end of the first cell is significantly greater than the nozzle width W. This effect explains why in Ref. 3 the cylindrical resonator disk gap had to be open much larger than the nozzle width in order to achieve the peak performance.

For application to cylindrical resonator design, the Mach number distribution along the r axis of the radially directed jet is of primary interest. Experimental results obtained from measurement of the shock angle of a 15-deg cone probe are given in Fig. 5 at a pressure ratio of 3.76. From the figure, it can be seen that the characteristics solution predicts the radial Mach number distribution reasonably well.

It may be concluded that the method of characteristics yields a fairly accurate description of the radially directed jet, at least as far as the first cell length and Mach number distributions are concerned. As might be anticipated on physical grounds, the first cell length and the jet width at the end of the first cell are significantly greater than for the two-dimensional case. Consequently, where theoretical estimates of the jet structure for radially excited resonators are required, the present technique should be employed. Otherwise, the use of two-dimensional methods could lead to incorrect cavity dimensions and inefficient resonator performance. Additional work will be done to find the relationship among the cavity radius, cell length and end-width, nozzle width, and pressure ratio.

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References

[1] Przirembel, C. E. G. and Fletcher, L. S., "Aerothermodynamics of a Simple Resonance Tube," *AIAA Journal*, Vol. 15, Jan. 1977, pp. 101-104.

[2] Smith, T. J. B. and Powell, A., "Experiments Concerning the Hartmann Whistle," Univ. of California, Berkeley, Calif., Rept. 64-62, 1964.

[3] Wu, J. H. T., Ostrowski, P. P., Neemeh, R. A., and Lee, P., "Experimental Investigation of a Cylindrical Resonator," *AIAA Journal*, Vol. 12, Aug. 1974, pp. 1076-1078.

Flutter of an Elastic Plate under Tension

E. H. Dowell*
Princeton University, Princeton, N.J.
and
C. S. Ventres†
Bolt, Beranek and Newman, Inc., Cambridge, Mass.

THIS problem is reconsidered and a new interpretation of existing results presented. For simplicity, two-dimensional aerodynamic flow over a one-dimensional (infinitely wide) plate is considered. However, the neglect of finite width should not alter the qualitative nature of the results or the essential conclusions that are reached.[1] The primary purpose is to clarify the effect of variations in plate bending stiffness on the flutter of a plate under a given tension, and vice versa. The particular limit as the plate bending stiffness vanishes gives a membrane whose flutter behavior has led to much discussion in the literature. In this limit, for sufficiently large supersonic Mach numbers where quasi-static Ackeret aerodynamics may be used and coupled mode flutter would occur for a plate with finite bending stiffness, the theory predicts that no flutter will occur. Spriggs et al.[2] have investigated this limit carefully and shown that for small (but nonvanishing) bending stiffness flutter will occur when

$$\lambda\epsilon^3 > (2/3)^{3/2} \tag{1}$$

where

$\lambda \equiv 2qa^3/\beta D$
$\epsilon^3 \equiv (D/a^2N_x)^{3/2}$
a ≡ plate length
D ≡ plate bending stiffness
N_x ≡ tension
q ≡ dynamic pressure of aerodynamic flow
$\beta \equiv (M^2-1)^{1/2}$
M ≡ flow Mach number

Inequality (1) holds for $\epsilon \to 0$ (as we shall see for $\epsilon \lesssim 0.025$) when the tension-induced stiffness is much larger than the bending stiffness. Inequality (1) may be rewritten as[3,4]

$$2qD^{1/2}/\beta N_x^{3/2} > (2/3)^{3/2} \tag{1a}$$

which is independent of plate length a and, moreover, shows that as D becomes smaller a larger q is required to initiate flutter. Indeed, as $D \to 0$, the q required for flutter approaches infinity. This is the essence of the "membrane paradox."[2]

It should be noted that Spriggs et al.[2] have chosen to introduce the membrane stress σ_x, which is related to N_x by $N_x = \sigma_x h$, where h is the plate thickness. They further write the bending stiffness as $D = Eh^3/12(1-\nu^2)$, where E is the modulus of elasticity and ν is Poisson's ratio. If one holds σ_x constant, then inequality (1) is independent of h, and, as $h \to 0$ [and hence the bending stiffness $D \equiv Eh^3/12(1-\nu^2) \to 0$], a finite q is obtained from inequality (1) which is required to initiate flutter. In this sense, there is no "membrane paradox." Although the foregoing is perfectly correct mathematically, the physical choice of maintaining constant σ_x rather than constant N_x seems less likely to be appropriate in most applications. One would expect a constant N_x to be imposed in most cases (independent of h), and this is the approach followed here.

Further insight into this remarkable result can be gained by considering the flutter boundary in terms of λ and ϵ for the full range of ϵ. For $\epsilon \to \infty$, it is well known from the work of Hedgepeth[5] and subsequent authors that flutter occurs when (for simply supported leading and trailing edges)

$$\lambda > 343.3 \tag{2}$$

For intermediate ϵ, the results may be presented graphically using the numerical data of Dugundji,[6] Dixon,[7] and Spriggs et al.[2] In Fig. 1, the flutter boundary is shown in terms of λ vs $1/\epsilon$. This presentation is appropriate for a plate of a given bending stiffness where the effect of tension on the flutter boundary is being considered. As can be seen, as the tension $1/\epsilon$ increases,

$$1/\epsilon \equiv (N_x a^2/D)^{1/2}$$

the dynamic pressure required for flutter to occur, $\lambda \equiv 2qa^3/\beta D$, also increases monotonically. Here, because D is held fixed and N_x varied, no membrane paradox is evident.

Now consider the same data presented in Fig. 2 in terms of $\lambda\epsilon^2 \equiv 2qa/\beta N_x$ vs $\epsilon \equiv (D/N_x a^2)^{1/2}$. Also shown are the asymptotic results given by Eqs. (1) and (2). Here the tension N_x is considered fixed and the bending stiffness D is varied. The results shown in Fig. 2 display a result at variance with one's intuition. There is a minimum in the dynamic pressure required for flutter at an associated value for bending stiffness, $\epsilon \simeq 0.1$. For values of bending stiffness larger or smaller than $\epsilon \simeq 0.1$, the dynamic pressure required for flutter is in-

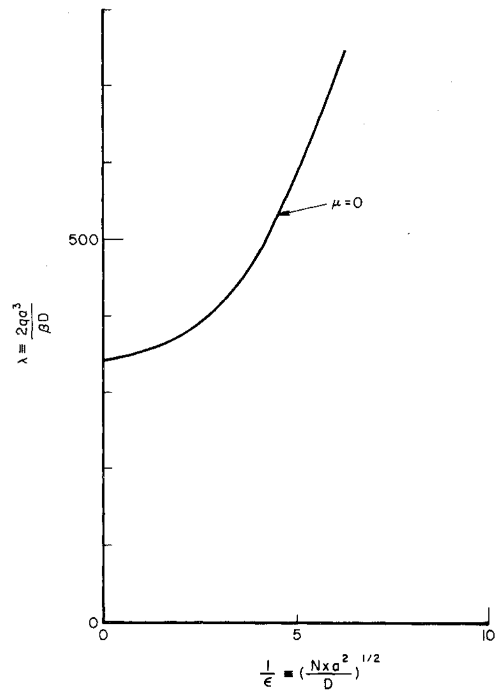

Fig. 1 Flutter boundary in terms of λ vs 1/ε.

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*Professor, Department of Aerospace and Mechanical Sciences.
†Senior Scientist.